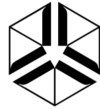

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A Predicative Semantic Model for Integrating UML Models

Jing Yang, Quan Long, Zhiming Liu and Xiaoshan Li

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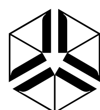

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P.O. Box 3058
Macau

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Abstract

This paper presents a predicative semantic model for integrating models from UML class diagrams and sequence diagrams. The integrated model is used for dealing with consistency problems of UML class diagrams and sequence diagrams. We also define the notion of consistent refinement of these integrated models.

Jing Yang is a fellow of UNU-IIST from Guizhou University, Guiyang, China, where she is an Associate Professor. Her research interests include the semantics of interaction diagrams in UML and its refinement and wavelet theory and its application. E-mail: yj@iist.unu.edu.

Quan Long is a fellow of UNU-IIST from Peking University, Beijing, China, where he is a doctorial candidate. His research interests include Software development process and formal techniques for Object Oriented, UML and Component based Software. E-mail: longquan@iist.unu.edu.

Zhiming Liu is a research fellow at UNU/IIST, on leave from Department of Computer Science at the University of Liecester, Liecester, England where he is lecture in computer science. His research interests include theory of computing systems, including sound methods for specification, verification and refinement of fault-tolerant, real-time and concurrent systems, and formal techniques for OO development. His teaching interests are Communication and Concurrency, Concurrent and Distributed Programming, Internet Security, Software Engineering, Formal specification and Design of Computer Systems. E-mail: Z.Liu@iis.unu.edu.

Xiaoshan Li is an Associate Professor at the University of Macau. His research areas are interval temporal logic, formal specification and simulation of computer systems, formal methods in system design and implementation. E-mail: xsl@umac.mo.

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1 Introduction

In a UML-based development process, such as the RUP [24, 14], several kinds of UML models are used to represent and analysis the artifacts created in a certain phase of the system development, which reflect the multiple views of UML:

- *Static view*: class diagram for static analysis.
- *Interactive view*: sequence diagrams and collaboration diagrams for interactions between objects.
- *Behavioral view*: state machines for dynamic behavioral specification and validation.
- *Functional view*: OCL [26] specifications for functionalities of objects.

Under the multiple views of UML, the developers can decompose a software design into smaller parts of manageable scales. However, several challenging issues inevitably arise from such a multi-view approach [21]:

- Consistency: the models of various views need to be syntactically and semantically compatible with each other (i.e. horizontal consistency)[20, 9, 1].
- Transformation and evolution: a model must be semantically consistent with its refinement (i.e. vertical consistency)[9, 1].
- Traceability: a change in the model of a particular view leads to corresponding consistent changes in the models of other views.
- Integration: models of different views need to be seamlessly integrated before software production.

Consistency checking and formal analysis of UML models have been widely studied in recent years [6, 4, 9, 23, 1]. A formal semantic model is needed for precise and intensive treatment of the problems. The informal semantics of UML is deliberately left flexible and extendable in order to allow UML to be used for different purposes, such as for requirement analysis, refinement of designs, and for code generation and testing.

The majority of the existing works on formal support to UML-based development, e.g.[7, 2, 6, 5, 10, 23], focus on the formalization of individual diagrams and only treat the consistency of the models of one or two views. Another phenomenon in research on formal use of UML is that different communities intend to emphasize different notations and use the full or even extended power of, say sequence diagrams or state machines. This would lose the advantages of the multiple-view modelling. It also leads to the increase in the complexity of a certain kind of models and the reduction of the role that the other kinds of UML models can play. To our

knowledge, there is little work on consistent refinement of complete UML models of systems. A complete model of a system here means a family of models for the different views of the system.

This paper is towards the development of a semantic model of UML. The primary use of this model is for model integration, refinement and code generation [22]. The integration is based on the object-oriented specification language (OOL) defined in [13] that is designed for object-oriented system development in general. The refinement calculus for OOL [11, 13] will be used to define consistent refinement of UML models. The refinement process will preserve the consistency and the correctness of the system. The proposed techniques are also intended to support *model-driven development* [25] for executable UML models. As a starting work towards UML model integration and code generation, in this paper we only consider sequential software systems for which the UML class diagrams and sequence diagrams are powerful enough. The future work will extend this approach to deal with concurrent systems for which we need the other UML models, i.e. component diagrams, activity diagrams and statecharts.

The rest of the paper is organized as follows. Section 2 describes the theoretical basics of programming and presents an overview of OOL. Section 3 provides formalization of UML diagrams and system models using OOL semantics with the definition of model consistency. We show how a model is refined consistently in Section 4. Finally, concluding remarks are given in Section 5.

2 The Theoretical Basics

Our work is based on Hoare and He's *Unifying Theories of Programming*[12], in which a program or a program command is identified as a *design*, which is represented by a pair (α, P), where

- α denotes the set of variables of the program, called the alphabet of the design.
- P, called the contract of the design, is a predicate of the form

  $$p(x) \vdash R(x, x') \stackrel{def}{=} (ok \wedge p(x)) \Rightarrow (ok' \wedge R(x, x'))$$

  where

  - x and x' stand for the initial and final values of program variables x in α,
  - predicate p, called the *precondition* of the program, characterizes the initial states in which the activation of the program will lead its execution to termination,
  - predicate R, called the *post-condition* of the program, relates the initial states of the program to its final states, and
  - we describe the termination behavior of a program by the Boolean variables ok and ok', where the former is true if the program is properly activated and the latter becomes true if the execution of the program terminates successfully.

The semantics of a command c, $[\![c]\!]$, is an implication of the form $\mathcal{D}(c) \Rightarrow \mathit{Spec}$, where $\mathcal{D}(c)$ defines the well-definedness of command c, *Spec* is a design of the form: $p(x) \vdash R(x, x')$. A command c_1 *refines* c_2 iff $\forall x, x', ok, ok' \cdot ([\![c_1]\!] \Rightarrow [\![c_2]\!])$.

2.1 An Overview of OOL

OOL [13] is an object-oriented language with a rich variety of features including subtypes, reference types, visibility, inheritance, dynamic binding, polymorphism and local variable nested declarations. The language is designed for reasoning about object-oriented software at different levels of abstraction including specifications, designs and programs.

2.1.1 Object-oriented systems.

An object system (or program) S is of the form $\mathit{cdecls} \bullet P$, where *cdecls* is a *declaration* of a finite number of classes, and P is called the main method and is of the form $(\mathbf{glb}, c)$ consisting of a finite set **glb** of *global variables* with their types and a command c. P can be understood as the *main method* if S is taken as a Java program. We do not allow any direct access to class attributes in the main method P, thus except for manipulation on variables in **glb**, P can only call methods of classes in the declaration section. The other syntactic categories of OOL includes *class declarations*, *class declaration sections*, *commands* and *expressions*. We treat class declarations in the same way as commands and whose semantics are to be defined as designs in terms of preconditions and postconditions. The semantics of OOL is fairly intuitive. One can informally understand the meaning of a program as the execution of its Java counterpart, once it is compiled correctly. However, it is important to note that the validity of an expression $D(e)$ is defined and should be checked when it is evaluated. An expression e is valid only if the types and variables used in e are declared. The dynamic type of an expression is recorded during evaluation.

2.1.2 Expressions.

Expressions, which can appear on the right hand sides of assignments, are constructed according to the rules: $e ::= x \mid \mathit{null} \mid \mathit{self} \mid e.a \mid f(e)$ where *null* represents the special object of the special class *NULL* that is a subclass of all classes and has *null* as its unique object, *self* will be used to denote the active object in the current scope, $e.a$ is the a-attribute of e, and $f(e)$ represents the built-in expression.

2.1.3 Class declarations.

A declaration *cdecls* is of the form: *cdecls* := *cdecl* | *cdecls*; *cdecl*, where *cdecl* is a *class declaration* of the following form

$$
\begin{array}{l}
\textbf{Class } N \textbf{ extends } M\ \{\\
\quad \textbf{private} \quad U_1\ u_1 = a_1, \ldots, U_m\ u_m = a_m;\\
\quad \textbf{protected}\ V_1\ v_1 = b_1, \ldots, V_n\ v_n = b_n;\\
\quad \textbf{public} \quad W_1\ w_1 = c_1, \ldots, W_k\ w_k = c_k;\\
\quad \textbf{method} \quad m_1(\underline{T}_{11}\ \underline{x}_1, \underline{T}_{12}\ \underline{y}_1, \underline{T}_{13}\ \underline{z}_1)\{c_1\};\\
\qquad\qquad\qquad \cdots;\\
\qquad\qquad m_\ell(\underline{T}_{\ell 1}\ \underline{x}_\ell, \underline{T}_{\ell 2}\ \underline{y}_\ell, \underline{T}_{\ell 3}\ \underline{z}_\ell)\{c_\ell\}\\
\quad \}
\end{array}
$$

Note that

- N and M are distinct names of classes, and M is called the direct superclass of N.
- The **private** declaration declares the private attributes of the class, their types and initial values, and similarly, the **protected** and **public** declarations for the protected and public attributes.
- The **method** declaration declares the methods, their value parameters $\underline{T}_{i1}\ \underline{x}_i$, result parameters $\underline{T}_{i2}\ \underline{y}_i$, value-result parameters $\underline{T}_{i3}\ z_i$ and bodies c_i. We sometimes denote a method by $m(\underline{paras})\{c\}$, where $\underline{paras}$ is the list of parameters of m and c is the body command of m.
- A method body c_i is a command that will be defined later.

The above class declaration is *well-defined*, if N has not been declared before, the names of the attributes are distinct, the types of the initial values of the attributes match the types of the attributes, and the names of the parameters of each methods are distinct. A class declaration section is *well-defined* if all the class declarations, commands and expressions in it are well-defined.

2.1.4 Commands.

Our language supports typical object-oriented programming constructs.

$c ::=$	$skip \mid chaos \mid \textbf{var}\ T$ x=e	termination, abort, local variable declaration
	$\mid \textbf{end}\ x \mid c; c \mid c \lhd b \rhd c$	local variable undeclaration, sequence, conditional choice
	$\mid c \sqcap c \mid b * c \mid le.m(\underline{e}, \underline{v}, \underline{u})$	nondeterministic choice, iteration, method call
	$\mid le := e \mid C.new(x)$	assignment, creating a new object

where b is a Boolean expression, e is an expression, and le is an expression which may appear on the left hand side of an assignment and is of the form $le ::= x \mid le.a$ where x is a simple variable and a an attribute name. Most of the commands have their classic meanings in the imperative setting. We only explain the following commands which are typical to object-oriented programming.

- **An Assignment:** $le := e$ is well-defined when le and e are well-defined and the type of e is a subtype of the declared type of le. In the case of a simple assignment $x := e$, when the assignment is well-formed it only modifies x by assigning x with the value of e; In the case of an modification of an object's attribute, $le.a := e$ changes the a-attribute of the object that le refers to into the value of e.
- **A Method Call:** A method call $le.m(v, r, vr)$ is well-defined if le is non-null object and $m(x, y, z)$ is a method declared in the type of le. When it is well-defined, its execution assigns the values of the actual parameters v and vr to the formal parameters x and z of m of the active objects that le refers to, and then executes the body of the method under the environment of the class of the active object. After the execution of the body terminates, the value of the result and value-result parameters y and z are passed back to the actual parameters r and vr.
- **An Object Creation:** $C.New(x)$ is well-defined if C is a declared class. Its execution creates an object of class C with a fresh reference, attaches it to variable x, and attaches the initial values of the attributes of class C to the attributes of x too.

3 A Formal Syntax and Semantics of UML Models

In this section, firstly we will give the syntax of the class diagram and sequence diagrams. After that we will describe the requirement model and design model in our framework. Finally we will investigate the conditions on the consistent issues and present the OOL semantics of a consistent model.

3.1 Syntax of class diagrams

A class diagram Γ (see Fig.1 and 5) identifies the following information.

1. The first part provides the static information on classes and their inheritance relationships:
    - **CN**: the finite set of classes has always been identified.
    - **super**: this is the direct generalization relation over the set **CN**.

2. The second part describes the structures of classes. For each $\mathbf{C} \in \mathbf{CN}$, we use $\mathbf{attr}(\mathbf{C})$ to denote the set

   $$\{< a_1 : \mathbf{T}_1 >, \cdots, < a_m : \mathbf{T}_m >\}$$

   of primitive attributes of $\mathbf{C}$, where $\mathbf{T}_i$ stands for the type of attribute a_i of class $\mathbf{C}$.

3. The third part provides the information about associations among classes: the finite set in which elements are of the forms:

   $$< \mathbf{C}_1,\ m_1,\ Ass,\ m_2,\ \mathbf{C}_2 > \quad | \ (\mathbf{C}_1,\ m_1,\ Ass,\ m_2,\ \mathbf{C}_2)$$

   The first represents a direct association Ass from $\mathbf{C}_1$ to $\mathbf{C}_2$. The second notation indicates a undirect association Ass between classes $\mathbf{C}_1$ and $\mathbf{C}_2$. Here m_1 and m_2 are of the forms: $1 \mid 0..1 \mid \ * \ \mid 1..*$, indicating multiplicities respectively. We call undirect association conceptual association, and direct association design association.

4. For each $\mathbf{C} \in \mathbf{CN}$, *method*$\mathbf{C}$ identifies the set of all methods of class $\mathbf{C}$.

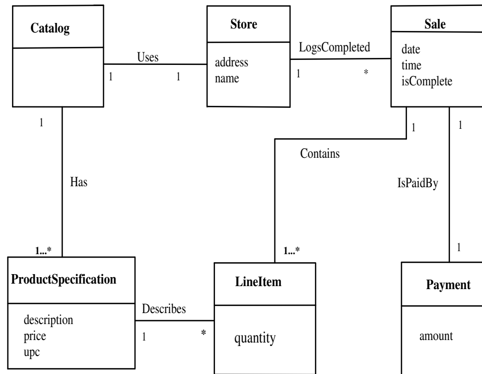

Figure 1: A conceptual class diagram for an automated checking out system in a shop

The main condition of the well-formedness of a class diagram is that the inheritance relation does not introduce cycles between classes. Also we do not deal with multiple inheritance in a class diagram. The other issues are mainly naming problems.

In our framework, the class diagram defines the *system state space* allowed by the application, and each system state encompasses some objects and all links among these objects. Therefore, the class diagram plays the role as declarations of classes, types and variables in a program. A system state is a well-typed state of the variables. We can therefore easily specify (or translate) a class diagram Γ into a declaration section $cdecl_\Gamma$ in OOL later.

3.2 Syntax of sequence diagrams

A sequence diagram consists of objects and ordered messages that describe how the objects communicate. An interaction occurs when one object invokes a method of another. We now

give the syntactic definition of sequence diagrams. We will allow *call back messages* in a sequence diagram (e.g. message "(5)" in Fig.2). The definition covers most features of UML 2.0 including combined fragments (except for the PAR one) [3], reference to other sequence diagrams and nested sequence diagrams. The object "$\underline{: B}$" in Fig.2 represents a nested sequence diagram.

A sequence diagram *SD* consists of two parts:

1. A sequence of objects: $\langle obj_1, obj_2, \cdots, obj_n\rangle$. Each object obj_i has the following structure:

   - Each object *obj* is associated with a **type**, denoted by **type**(*obj*), which is either a class name **C** in **CN** or a sequence diagram SD_1.
   - For each object *obj*, the property **multiob**(*obj*) equals *true* if *obj* is a multi-object (e.g. $\underline{: C}$ in Fig.2), otherwise **multiob**(*obj*) is *false*. Multi-objects represent one-to-many associations in a class diagram.
   - For each object *obj* there is a sequence of time-points $\langle p_1, p_2, \cdots, p_n\rangle$ which are totally ordered and represent the time points when an event occurs during the lifetime of the object. These points represent the ordering of messages sending and receiving, the combination fragments and the references to other sequence diagrams.

     We have a function *event* for each time-point p and *event*(p) describes what happens at time-point p. For each time-point p, *event*(p) can be one of the elements in the following set

     $$\{send,\ ack,\ receive,\ receiveack,\ option,\ loop,\qquad endfrag,\ ref,\ endref\}$$

2. A set *MSG* of messages: each message *msg* is one of the forms (src, m, tgt), (m, tgt) or (src, m) where

   - *src*, denoted by *source*(*msg*), is a pair (obj, p) of an object and a time-point, and $source(msg) = (obj, p)$ means that object *obj* is the source of the message that occurs at time-point p, and we use $(obj, p).object$ to denote *obj* and $(obj, p).point$ to denote p.
   - *tgt* is a pair (obj, p) of an object and a time-point, represented by *target*(*msg*) and
   - m, denoted by *method*(*msg*), is any command in OOL. Therefore, m can be a method call of the form $(ass, method())$ (sometimes it is simply written as $ass.m()$), that represents that $method()$ of the target object is called by the source object via the association *ass*. Also, m can be a command, such as a design, an assignment, or any composite command of other kinds, but we require in this case the source object and the target object must be the same. This represents the execution of an internal action of the object. Finally, a message can be a return signal and in this case m is denoted as **return**.
   - A message (m, tgt) represents an incoming message to the sequence diagram and in this case m must be a method call. A message (src, m) shows an outgoing message from the sequence diagram and in this case m must be **return**.

   We do not show **return** in the diagram.

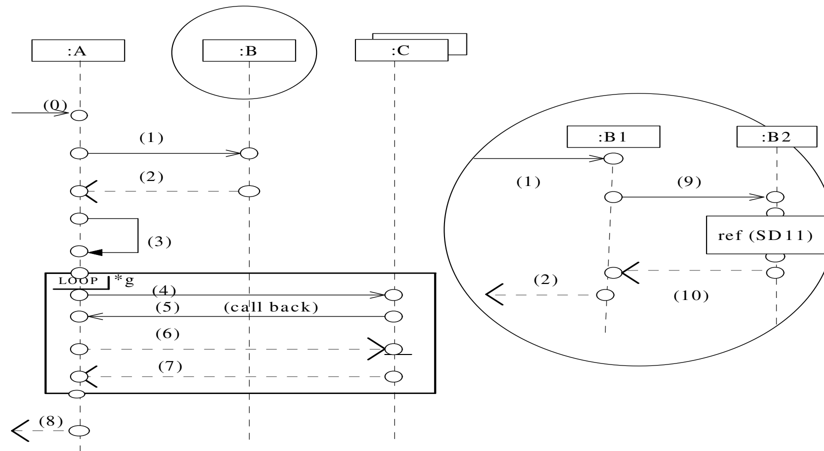

Figure 2: An example sequence diagram

Fig.2 is an example of a sequence diagram in which

- *event*(*p*) = *send* shows a message is sent from this position of the current object. Similarly, *receive* means a message is reached at this position of the object. The message between a *send* point and a *receive* point will be drawn in a solid line with arrow to the *receive* point in the graph. For example, the message "(1)" and "(4)" in Fig.2.
- *ack* and *receiveack* points are used to denote a **return** message which are drawn in dotted line with open arrowhead back to the lifeline of the source object. For example, the message "(2)" and "(7)" in Fig.2.
- If an *event* of a point is *option* or *loop*, it will be equipped with another function, **guard**, which maps the point to its *guard* that is Boolean expression of the source object's attributes.

  The option combination fragment is used to represent a sequence that will be executed if the guard condition holds. An option combination fragment is used to model a "conditional choice" statement. The loop combination fragment is used to represent a repetitive sequence. The body of the fragment will continue to be executed repetitively until the guard condition becomes false. The *event*(*p*) = *endfrag* represents the end of a combination fragment.
- *event*(*p*) is *ref* means that from point *p* the current sequence diagram begins to call another sequence diagram and *endref* represents the end of the call. A *ref* point will be equipped with a **name** representing the sequence diagram it calls.

3.3 Well-formedness of sequence diagrams

We need to ensure that a sequence diagram is well-formed. The well-formedness is concerned with the following conditions:

- For each message *msg* in the sequence diagram, the *event* of the source point of *msg* must be a *send* or *ack* and the *event* of target point of *msg* must be a *receive* or *receiveack*, respectively.
- If a point p_1 represents the beginning of combined fragments, i.e. *loop* or *option*, there must be one and exactly one corresponding *endfrag* point p_2 on the same object such that p_2 is later than p_1.
- For a point p_1 with $event(p_1) = ref$, there must be point p_2 on the same object such that $event(p_2) = endref$ and p_2 is later than p_1. The well-formedness of the referred sub-sequence diagram is checked recursively.
- If *obj* is a nested sequence diagram, then for every *matched* pair of sending and returning messages $(src, m, (obj, p_1))$ and $((obj, p_2), m, tgt)$, there is a corresponding matched pair of messages (m, tgt_1) and (scr_1, m) of source-less (incoming) message and target-less (outgoing) message in the sub-sequence diagram $\mathbf{type}(obj)$. The order of these message is preserved in the sub-sequence diagram and the sub-sequence has to be well-formed.

The well-formedness of a sequence diagram has also to ensure the sequence diagram indeed represents a scenario of method calls. This means that 1). order of the message sending and receiving must be consistent, and for all messages from the same object, the earlier it is sent the earlier it is received by the target object; 2). if a message *msg* invokes message msg_1, then msg_1 must return before *msg* does.

For a sequence diagram, let *action* be the set of all its messages, combined fragments, and referred sub-sequence diagrams. For actions $action_0$ and $action_1$ from the same object, we use the notation

$$returned(action_0, action_1)$$

representing that the execution of $action_0$ is finished before the execution of $action_1$. In particular, if $action_0$ is a message with a method call $meth_0$, then its corresponding **return** message must be received by the object before the the execution of $action_1$.

3.3.1 Definition 1 (Directly invoke)

Let msg_0 be a message in a sequence diagram SD.

- **message.** Message msg_0 *directly invokes* msg_1, denoted by $Invoke(msg_0, msg_1)$ if $target(msg_0).object = source(msg_1).object$ and $target(msg_0).point$ is the latest point in the set

  $$\{p | p \in target(msg_0).Points \wedge p < source(msg_1).point \wedge \neg returned(msg_0, msg_1)\}$$

  where $target(msg_0).Points$ is the time-points of the target object of msg_0 in SD.

- **fragment.** Let *freg* be a combined fragment and (obj, p) is the beginning point for *freg*. We say msg_0 *directly invokes freg*, denoted by $Invoke\ (msg_0, freg)$, if $target(msg_0).object = obj$ and $target(msg_0).point$ is the latest point in the set

  $$\{t | t \in target(msg_0).Points \wedge t < p \wedge \neg returned(msg_0, freg)\}$$

- **ref.** Let SD_1 be another sequence diagram and $rf = (obj, p)$ is the *ref* point where SD is called. We say msg_0 directly invokes *rf*, denoted by $Invoke(msg_0, rf)$, if $target(msg_0).object = rf.object$ and $target(msg_0).point$ is the latest point in the set

  $$\{p | p \in target(msg_0).Points \wedge p < rf.point \wedge \neg returned(msg_0, rf)\}$$

3.3.2 Definition 2 (Directly follow)

Let msg_0, msg_1 be two messages, $source(msg_0) = source(msg_1)$. We say msg_1 *directly follows* msg_0, denoted as $Follow(msg_0, msg_1)$, if $source(msg_1).point$ is the smallest element of the set

$$\{p | p \in source(msg_0).Points \wedge p > source(msg_0) \wedge returned(msg_0, msg_1)\}$$

The directly follow relationship between other actions (fragments or the references to other sequence diagrams) is similar to **Definition 2**.

3.4 Requirement models in UML

As in [15, 17], the development cycle of a software system starts with the construction of a *requirement model RM.* In UML, requirements are captured and described by a conceptual class diagram and some use cases. Conceptual class diagram is a class diagram in which every class has no method, and the associations are conceptual (i.e. undirect). Informally, such requirement model consists of a number of UML models, including a *conceptual class diagram* Γ_r, a *use-case diagram* U_r, a family Δ_r of *use-case* (or *system*) *sequence diagrams* (one for each use case), and some *activity diagrams* if concurrency is concerned. In this paper, we do not consider

concurrency[1]. We can thus denote the requirement model by a triple $RM = \langle \Gamma_r, U_r, \Delta_r \rangle$. Each use case U is modelled as a use-case controller class *U-Controller* (see Fig. 3).

$$
\begin{array}{ll}
\textbf{Class}\ \ U\text{-}Controller\ \{ & \\
\textbf{private}\ \underline{T}\ \underline{x}; & \\
\ \textbf{method} & op_1(< T_{11}\ x_1 >, < T_{12}\ y_1 >, <>)\{c_1\}; \\
 & \cdots; \\
 & op_n(< T_{n1}\ x_n >, < T_{n2}\ y_n >, <>)\{c_n\} \\
\} &
\end{array}
$$

where the attributes $\underline{x}$ may include state control variables which are private to the controller class. For each method of the form $op_i(< T_{i1}\ x_i >, < T_i\ y_i >, <>)\{c_i\}$, x_i is a list of value parameters, y_i a list of result parameters, and c_i is a command.

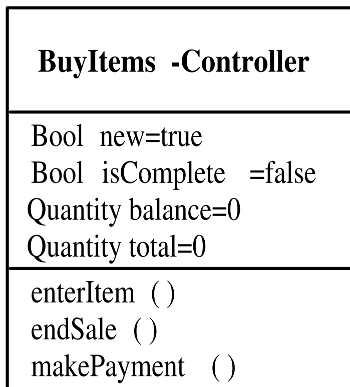

Figure 3: The controller class for BuyItems

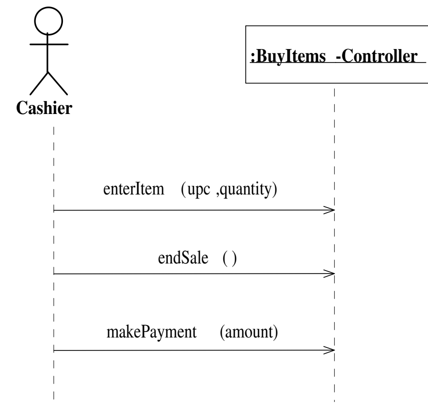

Figure 4: The use-case sequence diagram for use case BuyItems

This formalization implies that all attributes and associations in other classes are directly visible to all the use-case controller classes. The use-case diagram also provides the information about the design associations among the use-case controller classes. If use case U_1 includes use case U_2 in the use-case diagram, then there is an association from U_1-*Controller* to U_2-*Controller*. Use-case sequence diagrams Δ_r (see Fig.4), describe the the interaction between the actors and the system. Following the facade controller pattern in [15], for the operations in a use-case sequence diagram we declare the signatures in the corresponding use-case controller class. In a use-case sequence diagram, the receiver of each message is an object of the use-case controller class, and the sender is either an actor or an object of another use-case controller class.

Therefore, for each use case U, *U-Controller* declares the operations that appear in the use-case sequence diagram and the body of each method in *U-Controller* is defined from the system sequence diagram of the *U-Controller* class. Following the idea above, given a UML requirement

[1]It is often not recommended to consider concurrency at this early stage in an iterative development process.

model $RM = \langle \Gamma_r, U_r, \Delta_r \rangle$, the *normal form specification* in OOL for RM is:

$$[\![RM]\!] \stackrel{def}{=} cdecls_{\Gamma_r}; U_1\text{-}Controller; \ldots; U_n\text{-}Controller$$

U_i, $i = 1, \ldots, n$, are the use cases of RM.

We will give the details of the meaning of $[\![\cdot]\!]$ later.

3.5 Design models in UML

In UML, a design model DM should consist of a *design class diagram* Γ_d (see Fig.5), a family Δ_d of *object interaction* or *sequence diagrams*, at least one for each method in *U-Controller*. We can thus define a design model as an ordered couple $DM = \langle \Gamma_d, \Delta_d \rangle$.

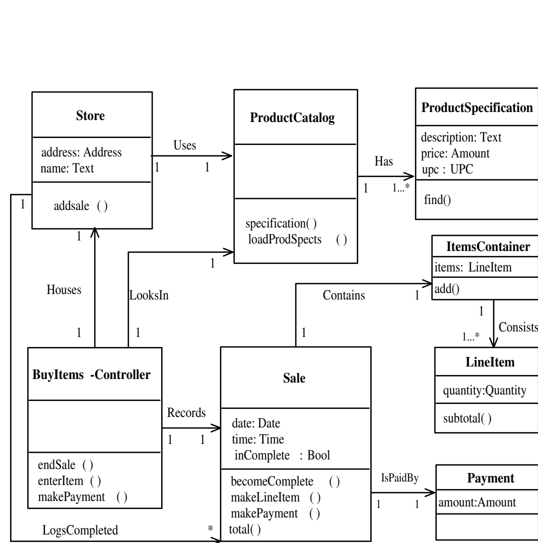

Figure 5: A design class diagram for use case BuyItems

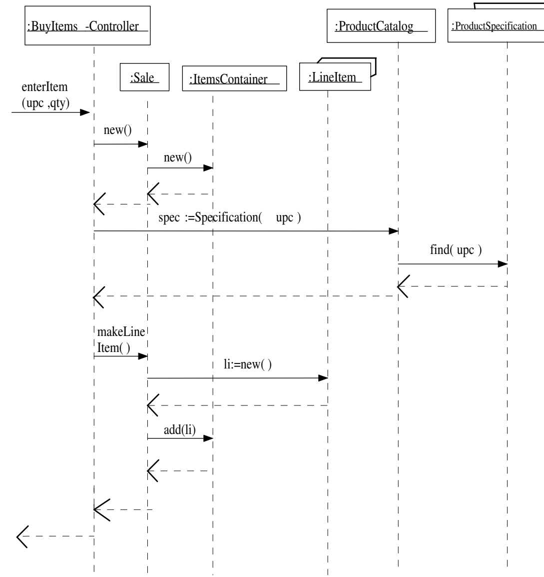

Figure 6: Sequence diagram for *enterItem*()

Object sequence diagrams Δ_d operate on the design class diagram Γ_d. In general sequence diagrams do not contain many details for describing the functionality of the system. In an informal UML-based development, other means, such as textual description, is used to describe the functionality of the system. In our formal framework, we provide formal specification of the body of the methods that will ensure the behavior required by the sequence diagrams.

Therefore, a design model *DM* is also specified as the *normal form specification* in OOL:

$$[\![DM]\!] \stackrel{def}{=} cdecl_1; \cdots ; cdecl_n$$

In next subsection we will give the details of the definition of $[\![\cdot]\!]$.

3.6 OOL Semantics for UML Models

In this subsection we will give the OOL semantics for a UML model. A UML model is an ordered couple $\langle \Gamma, \Delta \rangle$. If it is a requirement model, then $\Gamma = \Gamma_r \cup U_r, \Delta = \Delta_r$; if it is a design model, then $\Gamma = \Gamma_d, \Delta = \Delta_d$.

The semantics of a UML model $\langle \Gamma, \Delta \rangle$, is a sequence of class declarations in OOL, denoted by $[\![\langle \Gamma, \Delta \rangle]\!]$.

Before giving the semantics from a class diagram and a set of sequence diagrams, we need to ensure that these diagrams are consistent.

For a well-formed class diagram and well-formed sequence diagrams, we give the following items as the definition of consistency. A violation of any of them will be considered as an inconsistency.

- **Association.** For each $msg \in MSG$, there must be a corresponding association in the class diagram. Notice, this is static as it cannot ensure that the object which is sending the message in a particular state during the execution is currently associated with the target object of the message.
- **Class Name.** For the above-mentioned association, each of the two related classes in the class diagram must have the same name with the object related to *msg* in the sequence diagram.
- **Method.** Each method signature in the sequence diagram must be the same as the one in the class diagram. Furthermore, if $m()$ is the method of a message sent from $\underline{: C}$ to $\underline{: D}$ in the sequence diagram, then $m()$ must be a method of class D in the class diagram.
- **Attribute.** The variables used in the guard of a message should be directly accessible by the source object.
- **Multiplicity.** If an association in class diagram is one-to-many, the corresponding object in the sequence diagram must be a multi-object. Notice that multiplicity and other general class invariants should be ensured by the design of the sequence diagram, not by the consistency checking.

Now we give the semantics of consistent model $\langle \Gamma, \Delta \rangle$, denoted by $cdecl_{\langle \Gamma, \Delta \rangle}$, as follows:

- **Class.** A class $\mathbf{C}$ in Γ is declared as a *skeleton* of class declaration *cdecl* in $cdecl_{\langle \Gamma, \Delta \rangle}$ with their attributes and method signatures. All attributes are declared to be public.
- **Association.** For association *role name* a from $\mathbf{C}_i$ to $\mathbf{C}_j$ in Γ, if the multiplicity of $\mathbf{C}_j$ is 0..1 or 1, $cdecl_i$ for $\mathbf{C}_i$ has an attribute a with the type $\mathbf{C}_j$; and if the multiplicity of $\mathbf{C}_j$ is $1..*$ or $*$, $cdecl_i$ for $\mathbf{C}_i$ has an attribute a of type $\mathbf{C}_j$ and an attribute *a-set* of type of the powerset $\mathbb{P}\mathbf{C}_j$.
- **Constraints:** Constraints, such as invariants, multiplicity and aggregation, are specified in terms of pre-post conditions of methods in OOL.
- **Reference attributes:** For a sequence diagram $SD \in \Delta$, a method of an object of type $\mathbf{C}_1$ is called by an object of type $\mathbf{C}_2$, there will be an attribute with the reference type of $\mathbf{C}_1$ in the class declaration for $\mathbf{C}_2$.
- **Method bodies:** The method bodies in each class will be determined by the scenarios of method call in sequence diagrams Δ.
  - *Directly invoked actions:* For a message $msg = (src, m, tgt) \in \Delta$, if m is a method signature, SD is a particular sequence diagram in which m is called, then the following sequence of actions, denoted by $body_{SD}$, is a path of the execution of an invocation of m;

    $$action_1; action_2; \cdots action_n$$

    where $Invoke(msg, action_1)$ and for each $i : 0 < i < n$, $Follow(action_i, action_{i+1})$, and no more actions directly follow $action_n$.
  - *Method appears in several sequence diagrams.* If a method m appears in several sequence diagrams, say, $SD_1, SD_2, \cdots, SD_n \in \Delta$ and

    $$body_{SD_1}, body_{SD_2}, \cdots, body_{SD_n}$$

    are the corresponding method bodies in these sequence diagrams. Then the body of m is the non-determined choice of them:

    $$m()\{\sqcap_{i=1}^{n} body_{SD_i}\}$$

The overall well-formedness of the class diagram and the sequence diagrams and their consistency are ensured by the *well-formedness* of the OOL specification [13] obtained from the above definition. This translation can be automated and we have designed an algorithm in [22] for this purpose.

3.6.1 Example 1

The model composed of the class diagram in Fig.5 and the sequence diagram in Fig.6 has the semantics in OOL as follows.

Class *Store* {
  public *Address address*, *Text name*, *ProductCatalog prod*,
    $\mathbb{P}$*Sale sli*, *Sale s*, *BuyItems-Controller buyctr*;
  method *addSale*()
}

Class *ProductCatalog* {
  public $\mathbb{P}$*ProductSpecification proli*, *ProductSpecification pro*;
  method *specification*(*UPC upc*, *ProductSpecification spec*){
      pro.find(*upc*)
      }
    loadProdSpects()
}

Class *ProductSpecification* {
  private *Text description*, *Quantity price*, *UPC upc*, $\mathbb{P}$*LineItem li*, *Lineitem l*;
  method *find*(*UPC upc*)
}

Class *BuyItems − Controller* {
  private *ProductCatalog p*, *Sale sale*;
  method *endsale*();
    enterItem(*UPC upc*, *Quantity qty*){
      sale := *Sale.new*();
      var *spec* := *p.specification*(*upc*);
      sale.makeLineItem(*spec*, *qty*)
      }
    makePayment(*Amount amount*, *Amount balance*)
}

```
Class Sale {
  public Date date, Time time, Bool inComplete, ItemContainer icont,
        ℙLineItem lset, LineItem lineItem, Payment p;
   method  new(){
                 lineitem := LineItem.new()
                 }
               makeLineItem(ProductSpecification spec, Amount total){
                 var li := lineItem.new();
                 icont.add(li)
                 }
               makePayment(Amount amount, Amount balance);
               becomeComplete();
               total()
}
```

```
Class LineItem {
  public Quantity quantity;
  method subtotal(Quantity quantity, Quantity price, Amount subtotal)
}
```

```
Class Payment {
  public Amount amount;
  method
}
```

```
Class ItemContainer {
  public ℙLineitem items, Lineitem item;
  method add(LineItem item, ℙLineItme items)
}
```

4 Model Refinement

Having given the definition of the semantics $[\![\cdot]\!]$ for UML models in the above section, now we can define the refinement and correct relationship between UML models.

Firstly, let us recall the following two refinement definitions in OOL[13].

Definition 3 (Refinement between object systems) Let S_1 and S_2 be object programs which the same set of global variables **glb**. S_1 is a refinement of S_2, denoted by $S_1 \sqsupseteq_{sys} S_2$, if the behavior of S_1 is more predictable and controllable than that of S_2.
$S_1 \sqsupseteq_{sys} S_2 \stackrel{def}{=} \forall \underline{x}, \underline{x}', ok, ok' \cdot (S_1 \Rightarrow S_2)$ *where* $\underline{x}$ *are the variables in* **glb**. This means the external behavior of S_1, i.e.the pair of pre- and post-global states, is a subset of that of S_2.

Definition 4 (Refinement between declaration sections) Let $cdecls_1$ and $cdecls_2$ be two declaration sections. $cdecls_1$ is a refinement of $cdecls_2$, denoted by $cdecls_1 \sqsupseteq_{class} cdecls_2$, if the former can replace the latter in any object system:

$$cdecls_1 \sqsupseteq_{class} cdecls_2 \stackrel{def}{=} \forall P \cdot (cdecls_1 \bullet P \sqsupseteq_{sys} cdecls_2 \bullet P)$$

where P stands for a main method $(\mathbf{glb}, c)$. Intuitively, it states that $cdecls_1$ supports at least the same set of service as $cdecls_2$.

Now we provide the definitions of model refinement and correctness.

Definition 5 (Model refinement) Let $\langle \Gamma_1, \Delta_1 \rangle$ and $\langle \Gamma_2, \Delta_2 \rangle$ be two UML models. $\langle \Gamma_2, \Delta_2 \rangle$ is refined by $\langle \Gamma_1, \Delta_1 \rangle$, denoted by $\langle \Gamma_1, \Delta_1 \rangle \sqsupseteq_{model} \langle \Gamma_2, \Delta_2 \rangle$if the former's semantics refines the latter's:

$$\langle \Gamma_1, \Delta_1 \rangle \sqsupseteq_{model} \langle \Gamma_2, \Delta_2 \rangle \stackrel{def}{=} [\![\langle \Gamma_1, \Delta_1 \rangle]\!] \sqsupseteq_{class} [\![\langle \Gamma_2, \Delta_2 \rangle]\!]$$

Definition 6 (Correct Design Model) A design model DM is correct with respect to the requirement model RM, denoted by $Correct(DM, RM)$ if it is a model refinement of RM:

$$Correct(DM, RM) \stackrel{def}{=} [\![DM]\!] \sqsupseteq_{model} [\![RM]\!]$$

As before-mentioned, the *normal form specifications* in OOL for requirement model and design model only concern the class declaration in an object program and its main method corresponds to the application program using services which are provided by the methods of the classes in the design model. Thus, in this article, we are only interested in refinement relation between declaration sections. We allow the following refinement rules to a declaration section within the UML framework.

1. Adding a class declaration: this corresponds to adding a class into the class diagram, the methods of the new class into sequence diagrams.
2. Introducing a *fresh* attribute to a class: this corresponds to adding a fresh attribute of primitive type to the class or adding a design association from the class to another.
3. Introducing inheritance. If none of the attribute of class N is appeared in class M or any superclass of M, we can make M a direct superclass of N.
4. Moving some attributes from a class to its direct superclass: if all subclasses of class N have a common attribute, the common attribute can be moved to class N from all of its subclasses.

5. Data encapsulation: Suppose class M has a public attribute, and no method of other classes accesses this attribute except those of subclasses of M, we change the visibility of this attribute from public to protected. Suppose class M has a protected attribute, and no method of its subclasses accesses this attribute, we change the visibility of this attribute from protected to private.

6. Adding a *fresh* method into a class: This approach allows us to add a method signature into the class in the class diagram, and add a sequence diagram.

7. Refining the body command of a method $m()\{c\}$ in a declared class. This may lead to the replacement of the subsequence diagrams involving with $m()$.

8. Moving a method from a class to its direct superclass if the method body does not access any protected or private attribute of the class.

9. Copying a method from a class to its direct subclass.

10. Delegating some tasks of a class to its associated classes. If a method of a class contains a sub-command that can be realized by a method of another class, we can replace that sub-command with a method invocation to the latter class. Notice the sequence diagrams involved the method should be refined too.

11. Removing unused attributes: for a private attribute, it can be dropped if it does not appear in any method of the class; for a protected attribute, it can be dropped if it does not appear in any method of the class and its subclasses; for a public attribute, it can be dropped if it does not appear in any method of any class.

12. Removing unused methods: if a method is not called by other method or the main method in the object program, the method can be removed.

4.0.2 Example 2

The conceptual class diagram in Fig.1, the system sequence diagram in Fig.4 together with the use-case diagram *BuyItems-Controller* in Fig.3 form a requirement model. We can apply the refinement calculus to this model for the design of the method *enterItem*() (similarly to methods *endSale* and *makePayment*) according to the design class diagram in Fig.5, then achieve the object sequence diagram in Fig.6 step by step:

1. *BuyItems-Controller* delegates the responsibility of creating a new sale to *Sale*,

2. *BuyItems-Controller* delegates the job of finding the specification using upc matching to *Catalog*, which delegates the job further to the multi-object with type of *ProductSpecification*,

3. *BuyItems-Controller* delegates the task of making a line item to *Sale*.

5 Conclusions and Future Works

This work is towards a formal foundation for component and object systems development based on the formal object-oriented specification notation OOL and its refinement calculus in [13, 11]. In OOL, *normal form specification* of an object system is a sequence of class declarations and a main method. Each class declaration consists of some attributes, method signatures and method-body definitions, which has corresponding notations in UML. In this paper, we focused on the formalization of UML class models and sequence diagrams in OOL. With this formalization, we can integrate these two kinds of UML models and carry out consistent refinement of these diagrams. The consistency conditions between these two kinds UML models are treated as the well-formedness conditions of their corresponding integrated specification in OOL.

Compared to most exitsing work, e.g. [6, 4, 9, 23, 1], our approach is also *transformational*. However, we also provide integration of UML models. Furthermore, because different sections in an OOL specification clearly corresponding different UML diagrams, the formal specification of the integrated model can be transformed back to UML diagrams, i.e. the transformation is reversible. This is very important, as this allows up to obtain refined UML diagrams from a refined OOL specification. Thus, this approach also support *re-engineering*.

In our formalization, the horizontal consistency is mainly static: the syntactic consistency can be checked statically using an algorithm, while the invariants can be verified with model-checking tools. Vertical consistency (of refinement) is mainly semantic. This has fully justified our use of predicates in meeting the challenge of UML formalization [7]. Fundamental techniques of program and data refinement can be applied to UML transformation. This also supports UML-based model-driven development [8, 21].

Our future work will also include the integration of activity diagrams (for concurrency) in the framework and the extension of OOL with the notation of components to support component diagrams in UML2.0. Tools have also been developed using this framework [27].

In our related works, general transition systems are introduced to provide an integrated model of conceptual class diagrams and use cases (without the treatment of sequence diagrams, state machines and use-case diagrams)[19]. A version of OOL without reference types is presented in [11] and used in [18] for the specification of the integrated model of [19]. Article [16] uses OOL for the specification of design class diagram and sequence diagrams, but without rules for model transformation.

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